

Electron-hole imbalance in superconductor-normal metal mesoscopic structures.

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We analysed the electron-hole or, in another words, branch imbalance (BI) and the related electric potential V_{imb} which may arise in a mesoscopic superconductor/normal metal (S/N) structure under non-equilibrium conditions in the presence of a supercurrent. Non-equilibrium conditions can be created in different ways: a) a quasiparticle current flowing between the N reservoirs; b) a temperature gradient between the N reservoirs and no quasiparticle current. It is shown that the voltage V_{imb} oscillates with the phase difference φ . In a cross-geometry structure the voltage V_{imb} arises in the vertical branch and affects the conditions for a transition into the π -state.

A few decades ago a great deal of interest was paid to the study of effects related to the so called branch imbalance (BI) (see Refs. [1, 2] and for example the reviews [3, 4]). The BI implies that populations of the electron-like and hole-like branches of the excitation spectrum in a superconductor or a normal metal are different. For example, the BI may arise in a superconductor near the S/N interface if a current flows through this interface and the temperature is close to T_c . The conversion of the quasiparticle current j_Q into the condensate current j_S occurs over a rather long length λ_Q called a BI relaxation length. Over this length populations of the electron-like and hole-like branches of the energy spectrum differ from each other. The difference between these populations is characterized by the distribution function $f_- = -(n_\uparrow - p_\downarrow)$; the function n_\uparrow is the distribution function of the electron-like excitations and $p_\downarrow(\epsilon) = 1 - n_\downarrow(-\epsilon)$ is the distribution function of the hole-like excitations. In the considered case of a spin-independent interaction one has $n_\uparrow = n_\downarrow$ and $p_\uparrow = p_\downarrow$. One can show that the function f_- differs from zero if $\text{div} j_Q \neq 0$. The non-zero distribution function f_- leads to the appearance of an electric potential V_{imb} in a superconductor (or in a normal metal) which can be expressed in terms of the function f_- (see below). The BI may also arise in a bulk superconductor. For example, if longitudinal collective oscillations with a finite wave vector q are excited in the superconductor, the BI arises because in this case $\text{div} j_Q = iq j_Q \neq 0$. When these modes are excited (they are weakly damped only near T_c), the quasiparticle current j_Q oscillates in a counter phase with the condensate current j_S , so that the total current remains equal to zero. These oscillations have been observed experimentally by Carlson and Goldman (Carlson-Goldman mode) [5] and have been explained theoretically in Refs. [6, 7]. Another example of a system, in which the BI arises, is a uniform superconducting film in the presence of a temperature gradient ∇T and a condensate flow. It was established experimentally [8] and theoretically [9, 10, 11] that in this case the BI has a magnitude which is proportional to $v_s \nabla T$, where v_s is the condensate velocity.

Recently there has been growing interest in the study of transport properties of S/N mesoscopic structures. Several interesting, phase-coherent effects have been observed in these structures. Among them one can mention the change of sign of the Josephson critical current I_c in a four-terminal S/N/S mesoscopic structure. If an additional dissipative current (or an applied voltage) between the N reservoirs in a S/N/S structure of a cross geometry exceeds a certain value, the current I_c changes sign [12] and the Josephson junction is converted into the π -state (a theory for this effect was developed in Refs. [13, 14, 15]).

In this Letter, we consider S/N mesoscopic structures under non-equilibrium conditions and study the BI which can arise in the N conductor if there is a finite phase difference φ between the superconductors in these structures. We briefly analyse effects arising due to the BI and discuss how some of them can be observed. To our knowledge these effects are missed in most papers on transport properties of mesoscopic S/N structures. In some cases the physics of the BI in mesoscopic S/N structures resembles that of the BI in a uniform superconductor under nonequilibrium conditions. However even in these cases the BI has its own specific characteristics. For example, the voltage V_{imb} , which is associated with the BI, is an oscillating function of the phase difference φ between the superconductors and it may appear even in the absence of a temperature gradient. Consider for example the structure in fig. 1a. We will show that the voltage $V_{imb}(y)$ (or the electric field) arises in the vertical wire regardless of how the system is brought out from the equilibrium state. One can apply a temperature gradient between the N reservoirs disconnected from the external circuit or one can pass a finite current between the N reservoirs as it was done in the experiment [12].

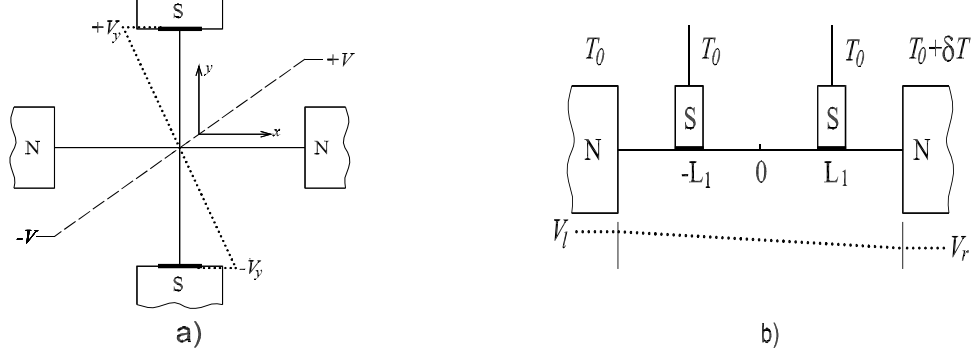


FIG. 1: Schematic view of the structures under consideration. Barriers or bad S/N interfaces are represented by thick lines. The dashed and dotted lines show the spatial distribution of the applied voltage (between the N reservoirs, fig. 1a) and the voltage caused by the charge imbalance. The electric potential in the superconductors is assumed to be zero.

When considering S/N/S structures, it is convenient to introduce two types of the distribution functions: f_{\pm} as it was done in Ref. [16]. The function f_{+} determines the order parameter in the superconductors and, for example, the condensate current in S/N/S structures. This function is related to the distribution functions of electrons $n_{\uparrow} = n_{\downarrow}$ and holes $p_{\uparrow} = p_{\downarrow}$: $f_{+} = 1 - (n_{\uparrow} + p_{\downarrow})$. The Josephson current in the N wire can be written in the form

$$j_s = (\sigma/2)W \int d\epsilon f_{+} J_s \quad (1)$$

where W is the cross-section area of the y -wire, the distribution function f_{+} , generally speaking, deviates from its equilibrium form $f_{eq} = \tanh \epsilon \beta$ and should be determined from the kinetic equation, here $\beta = 1/2T$. The function J_s is "a partial condensate current" and is expressed in terms of the condensate Green's functions $\hat{F}^{R(A)}$: $J_s = (1/4)Tr\{\sigma_z(\hat{F}^R \partial_y \hat{F}^R - \hat{F}^A \partial_y \hat{F}^A)\}$. This "current" does not depend on y . On the other hand, as follows from the conservation of the current, j_s can be written as a current through the S/N interface. This means in particular that "the current" J_s can be represented in the form $J_s = (r_s/L_y)Tr\{\hat{\sigma}_z(\hat{F}^R \hat{F}_s^R - \hat{F}^A \hat{F}_s^A)\}$, where the functions $\hat{F}^{R(A)}$ must be taken at the S/N interfaces; $r_s = R_y/R_s$, $R_y = \rho L_y$ is the resistance of the y -branch (per unit area) and R_s is the resistance of the S/N interface (per unit area). Under nonequilibrium conditions, which may have a different origin (non-equal temperatures of the S and N reservoirs, an additional current between the N reservoirs etc), the current given by eq. (1) is not equal to the Josephson current through the S/N interface because the last one depends not only on the value of f_{+} at the interface but also on the distribution function f_{S+} in the S reservoirs which is assumed to be equilibrium ($f_{S+} = f_{eq}$). This means that an electric field $E = -\partial_y V_{imb}$ arises in the y -branch even if there are no voltage differences neither between N reservoirs nor between the S reservoirs. The electric field drives the quasiparticle current which compensates the above mentioned difference between the condensate currents. The appearance of the electric field and the quasiparticle current means that strictly speaking eq. (1) does not describe the maximum current in the y -branch in the absence of the voltage difference between the S reservoirs (in most papers on this subject the maximum current was found from eq. (1)). In order to find the critical current in a non-equilibrium situation, one has to use a more general formula. However at low temperatures eq. (1) determines the critical current I_c with a good (exponential) accuracy.

In the present paper we consider S/N mesoscopic structures of two configurations (see fig. 1). In simple limiting cases we find the distribution functions and analyse the BI arising under non-equilibrium conditions. The distribution functions f_{\pm} obey the kinetic equations [16] which for the structure shown in fig. 1b can be written in the form (see for example [17])

$$L \partial_x [M_{\pm} \partial_x f_{\pm}(x) + J_S f_{\mp}(x) \pm J_{an} \partial_x f_{\mp}(x)] = r_S [A_{\pm} \delta(x - L_1) + \bar{A}_{\pm} \delta(x + L_1)]. \quad (2)$$

where all the coefficients are expressed in terms of the retarded (advanced) Green's functions: $\hat{G}^{R(A)} = G^{R(A)} \hat{\sigma}_z + \hat{F}^{R(A)}$; $M_{\pm} = (1 - G^R G^A \mp (\hat{F}^R \hat{F}^A)_1)/2$; $J_{an} = (\hat{F}^R \hat{F}^A)_z/2$; $J_s = (1/2)(\hat{F}^R \partial_x \hat{F}^R - \hat{F}^A \partial_x \hat{F}^A)_z$; $A_{\pm} = (\nu \nu_S + g_{1\mp})(f_{\pm} - f_{S\pm}) - (g_{z\pm} f_{S\mp} + g_{z\mp} f_{\mp})$; $g_{1\pm} = (1/4)[(\hat{F}^R \pm \hat{F}^A)(\hat{F}_S^R \pm \hat{F}_S^A)]_1$; $g_{z\pm} = (1/4)[(\hat{F}^R \mp \hat{F}^A)(\hat{F}_S^R \pm \hat{F}_S^A)]_z$. The

parameter $r_S = R_1/R_S$ is the ratio of the resistance of the N wire $R_1 = \rho L_1$ and S/N interface resistance R_S ; the functions \bar{A}_- and \bar{A}_+ coincide with A_- , A_+ if we make a substitution $\varphi \rightarrow -\varphi$. We introduced above the following notations $(\hat{F}^R \hat{F}^A)_1 = Tr(\hat{F}^R \hat{F}^A)/2$, $(\hat{F}^R \hat{F}^A)_z = Tr(\hat{\sigma}_z \hat{F}^R \hat{F}^A)/2$ etc.; ν , ν_S are the density-of states in the N film at $x = L_1$ and in the superconductors. The functions $f_{S\pm}$ are the distribution functions in the superconductors which are assumed to have the equilibrium forms. This means that $f_{S+} \equiv f_{eq} = \tanh(\epsilon\beta_o)$ and $f_{S-} = 0$, because we set the potential of the superconductors equal to zero. We neglect branch imbalance in the superconductors assuming that the distribution functions f_{\pm} recover quickly their equilibrium forms in S due to a big size of the superconductors in comparison with the size of the S/N interface. In the case of the structure in fig. 1a the left-hand side of eq. (2) should be written down for each branch of the structure and be set equal to zero. At the S/N and N/N' interfaces we use the boundary conditions which are given by the right-hand side of eq. (2) (at the N/N' interfaces the index S should be replaced by the index N' and all the condensate Green's functions in N' should be set equal to zero). Consider the structure shown in fig. 1a. Eqs. (1) can be integrated once in each branch, and, for example, in the y -branch we obtain

$$M_{\pm} \partial_y f_{\pm}(y) + J_S f_{\mp}(y) \pm J_{an} \partial_y f_{\mp}(y) = J_{y\gtrless}. \quad (3)$$

where $J_{y\gtrless}$ are the total "partial currents" in the upper ($y > 0$) and lower ($y < 0$) parts of the y -branch of the N wire. The current $J_{y\gtrless}$ is a "vector" with the components (J_{y+}, J_{y-}) . At the crossing point the current conservation law takes place $J_{y>} + J_{x>} = J_{y<} + J_{x<}$. At the S/N interface the current $J_{y>}$ is related to the Green's functions at $y = L_y$

$$J_{y>} = (r_S/L_y) A. \quad (4)$$

In order to obtain the current through the y -wire, the "current" $J_{y>}$ should be substituted into the integrand of eq. (1 instead of $f_+ J_S$. We present the solutions for the distribution functions assuming first the weak proximity effect. This means that the amplitude of the condensate functions in the N wire should be small: $|F^{R(A)}| \ll 1$. In this case $F^{R(A)}$ can be easily found from the linearized Usadel equation (the solution for the structure shown in fig. 1b is presented in ref. [23]). As follows from the form of the functions $F^{R(A)}$, they are small if the condition $\varepsilon \gg \varepsilon_y r_S$ is satisfied, where the characteristic energy ε is equal to the Thouless energy $\varepsilon_y = D/L_y^2$ (in the case of the geometry in fig. 1b the Thouless energy is $\varepsilon_1 = D/L_1^2$) or to the temperature T . In the case of the weak proximity effect, a solution for the kinetic equations (1) also can be easily found with the help of expansion in the parameter r_S (for the case $r_S > 1$ eq. (2) was solved numerically in Ref. [14]). We consider two types of the nonequilibrium situation: a) the temperatures of all the reservoirs are the same, but the electric potentials at the left and right N reservoirs are $\pm V$ (a current flows between these reservoirs); b) no current between the N reservoirs, but the temperatures of the left and right N reservoirs are different, so that the distribution function in the right (left) reservoirs $F_{r,l}$ is equal to: $F_{r,l} = \tanh(\epsilon\beta_{r,l})$. In the main approximation we find

$$\begin{aligned} a) f_+(x) &= F_{V+}; f_+(y) = F_{V+} \\ b) f_+(x) &= (F_l + F_r)/2 + (x/L_x)(F_r - F_l)/2; f_+(y) = (F_l + F_r)/2 \end{aligned} \quad (5)$$

and

$$\begin{aligned} a) f_-(x) &= (x/L_x)F_{V-}; f_-(y) = r_S(y/L_y)g_{z-}(f_{eq} - F_{V+}); \\ b) f_-(x) &= 0; f_-(y) = r_S(y/L_y)g_{z-}[f_{eq} - (F_r + F_l)/2] \end{aligned} \quad (6)$$

Knowing the distribution function f_- , we can calculate the electric potential in the y -branch V with the help of the formula [3, 16]

$$eV(y) = (1/8)Tr \int d\epsilon \hat{G}(\epsilon, y) = (1/2) \int d\epsilon v(\epsilon) f_- \quad (7)$$

where $\hat{G} = \hat{G}^R \hat{f} - \hat{f} \hat{G}^A$ is the Keldysh component of the matrix Green's function, $\hat{f} = \hat{1}f_+ + \hat{\sigma}_z f_-$ is the matrix distribution function, $v(\epsilon) = (1/4)Tr(\hat{\sigma}_z(\hat{G}^R - \hat{G}^A))$ is the density-of-states in the N wire. We easily find from eqs. (6-7) that the potential $V(y)$ is an odd function of y and the electric field $E(y)$ is an even function of y . We also see that the electric field arises only if the phase difference between the superconductors is not zero; otherwise the function g_{z-} is zero. The field E or the potential $V(y)$ oscillate with increasing phase difference φ . As is seen from eqs. (6-7),

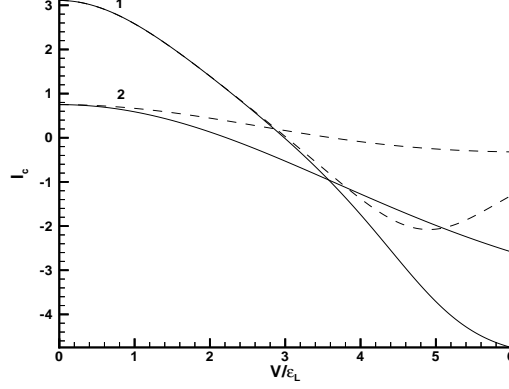


FIG. 2: The dependence of the critical current I_c (in arbitrary units) on the applied voltage V for two temperatures: $T = 0.5\epsilon_L$ and $T = 2\epsilon_L$, where $\epsilon_L = D/L^2$, $L_y = 0.3L$. The value ϵ_L is chosen in such a way that $\Delta(0) = 5\epsilon_L$. The solid lines are calculated from the correct eq. (4) and the dashed lines are calculated from the approximate, widely used eq. (1). The difference between these curves determines the voltage V_y (divided by the S/N interface resistance) at the S/N interface.

the electric field arises regardless of the origin of the non-equilibrium state: the function f_+ may deviate from the equilibrium function f_{eq} if a finite current flows between the N reservoirs or if the temperature of the N reservoirs differs from the temperature of the S reservoirs. In the considered case of a weak proximity effect the conductance G between the N reservoirs decreases with increasing the phase difference φ (at small φ). We also have calculated the critical (maximum) current using the correct expression eq. (4) and the approximate one eq. (1) (see fig. 2).

We see that the difference between two curves is significant if the temperature is not low. The difference between the critical currents is determined by the quasiparticle current $j_Q = \sigma E$. This follows directly from eq. (3) if one takes into account that in the main approximation $M_- \cong 1$ and the last term on the right-hand side can be neglected. It is worth noting that $f_- \neq 0$ only in the normal wire. In the superconductors the function f_- and therefore the voltage V are assumed to be zero, so that at the S/N interface there is a voltage drop from a finite value of V in the N wire to zero in the superconductor.

We also considered another limiting case when the proximity effect is not weak, that is, the condensate function in the N wire $|\widehat{F}|$ is not small. The obtained results qualitatively are similar to those which have been established for the weak proximity effect. The only difference is that the conductance in this case increases with increasing phase difference φ . Different behaviour of the conductance G as a function of φ was studied in Refs. [18] where a transition from a decreasing to increasing dependence (at small φ) of $G(\varphi)$ was obtained by varying the applied voltage. In a recent paper [19] a similar transition (obtained by varying the temperature) was studied in detail both experimentally and theoretically. Contrary to Refs. [18] it was assumed in Ref. [19] that the S/N interface is perfectly transparent. A good agreement between theoretical results and experimental data was obtained.

At last we consider a specific thermoelectric effect arising in the structure in fig. 1b. This effect was measured recently in mesoscopic S/N structures [20, 21]. As established in [23], if temperatures of the normal reservoirs are different, a voltage arises between these and superconducting reservoirs (the S reservoirs have the same electric potential because they are connected with a superconducting loop). The origin of this voltage which can be called thermoemf is completely different from the ordinary thermoemf in S/N/S junctions studied in Refs. [22]. In the last case the thermoemf appears due to the ordinary thermoelectric component of the quasiparticle current which is neglected here. In the limit of the weak proximity effect we have calculated the distribution functions f_{\pm} and the voltages $V_{l,r}$ caused by the temperature difference between the N reservoirs; here $V_{l,r}$ are the electric potentials at the left and right N reservoirs, respectively (the electric potential at the S reservoirs is set to zero). We assumed that the temperature of the left N reservoir T_o coincides with the temperature of the S reservoirs and the temperature of the right N reservoir T is elevated: $T = T_o + \delta T$. The distribution functions can be easily found using an expansion in the parameter r_S . For the voltages $V_{\pm} = (V_r \pm V_l)/2$, we obtain from eq. (7)

$$\begin{aligned} eV_+ &= -\delta T(L_1/L) \int d\epsilon(\epsilon\beta)g_{z+}(\epsilon, L_1)f'_{eq}/ \int d\epsilon g_{1+}(\epsilon, L_1)f'_{eq}; \\ eV_- &= r_S\delta T(L_1/2L) \int d\epsilon(\epsilon\beta)g_{z-}(\epsilon, L_1)f'_{eq} \end{aligned} \quad (8)$$

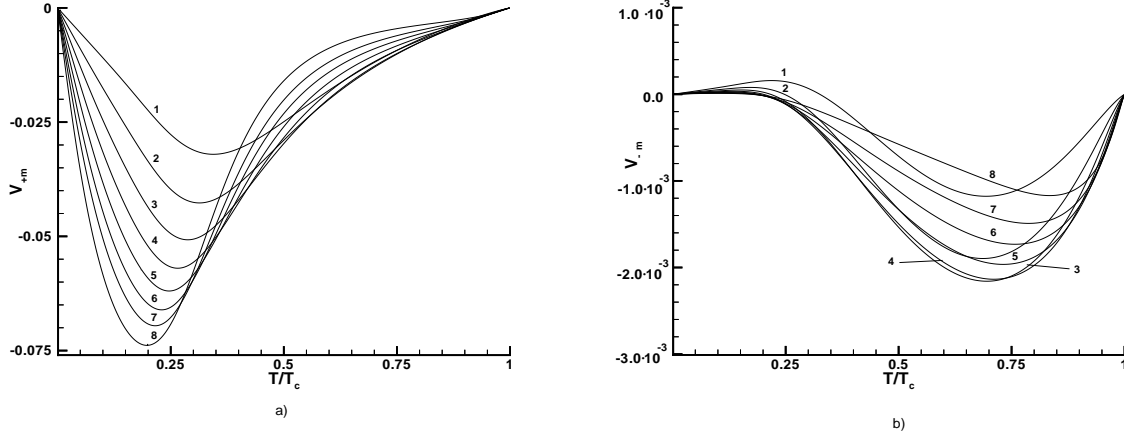


FIG. 3: The temperature dependence of the voltages $V_+ = (V_r + V_l)/2$ and $V_- = (V_r - V_l)$ caused by an extraordinary, phase-coherent thermoemf for different values of the parameter $\Delta(0)/\epsilon_L$: $\Delta(0)/\epsilon_L = 1.0; 1.4; 1.8; 2.2; 2.6; 3.0; 3.4$ and 4.0 for the curves 1 – 8, respectively.

where $f'_{eq} = \cosh^{-2}(\epsilon\beta)$. The expression for V_+ was obtained in Ref. [23]. Here we also obtained the voltage difference between the N reservoirs V_- . One can see that this voltage contains the small parameter r_S in comparison to the voltage V_+ . Both voltages are proportional to $\sin\varphi$, that is, they oscillate with increasing phase difference. The temperature dependence of the amplitudes of V_{\pm} are plotted in fig. 3.

We see that the maximum of the amplitude of V_+ is located at a lower temperature than the maximum of V_- . In principle this behaviour may lead to a nonmonotonic behaviour of the voltages V_r or V_l as a function of temperature. The change of the phase of the voltage oscillations with increasing phase φ may have the same origin as that in the case of the conductance [19], i.e. the change of the dependence $\varphi(H)$, where H is the external magnetic field. Our results can not be compared quantitatively with the recent experimental data [21] because the experimental structure corresponds to the case $r_S \approx 1$. We will analyse this more complicated case in a separate paper.

In conclusion, we have analyzed the branch imbalance effects in S/N mesoscopic structures. We have shown that in the structure in fig. 1a a voltage V_y related to the BI is set up if a current is driven through the x-axis or a temperature gradient exists between the N reservoirs. The voltage V_y is proportional to $\sin\varphi$, i.e. it oscillates with increasing phase difference φ . A similar voltage V_{imb} arises in the vertical wire if a temperature gradient δT exists between the N reservoirs. We also studied an unusual thermoelectric effect in the structure shown in fig. 1b. In this case voltages $V_{r,l}$ arise in the right and left N reservoirs (the electric potential at the superconductors is assumed to be zero) if there is a temperature difference between the normal reservoirs. These voltages are proportional to $\delta T \sin\varphi$ and they are not related to the ordinary thermoemf because we have ignored the small thermoelectric component in the quasiparticle current.

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